IB DP MATHEMATICS: ANALYSIS AN APPROACHES HIGHER LEVEL HANDBOOK

SAPPORO KAISEI SECONDARY SCHOOL

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INTRODUCTION

Mathematics provides us, as learners, with a unique and invaluable resource for explore, describe and express the world and its endless phenomena. In this sense it is similar to many of the natural sciences, yet in its own way entirely separate. Unlike the other empirical sciences, the certainty, finality and eternity of mathematical knowledge are what make it elegant and enjoyable to pursue, either for its own sake, or as a means of understanding the world.

Mathematics is driven from its core by the development of abstract ideas and notions and then connecting these to new ones. These concepts, in their rawest form, might not necessarily have any immediate real-world applications. However, the process of gathering ideas, creating connections, hypothesizing and then using sound logical reasoning to justify one's hypothesis is an invaluable transient skill, necessary in almost all areas of life, not just the academic disciplines.

Through this course of study students will hopefully able to engage with all elements of mathematical study. That is to say, understand both the necessity and value of mathematics in describing the world around us, but also the joy in deepening mathematical truth through reasoning. Furthermore, as students move through the course and into further education and/or employment, the will gain skills necessary to support success and allow them continue developing as life-long learners.

ABOUT THE

PROGRAMME

In General

The Mathematics Analysis and Approaches HL course has been designed based on an understanding that high-level analytical skills are becoming ever more important in a world where mathematics underpins many of the technological and industrial innovations currently taking place.

The course contains many elements of mathematics that are often found in a pre-university programme of study. This includes many topics that focus on the process of investigation, conjecture and proof. In conjunction with this, the course will develop in students' technological fluency. Students will use a variety of technology to construct, communicate and justify mathematical ideas and arguments.

Additionally, students will be required to undertake a short mathematical inquiry task called the "Mathematical Exploration". Whilst there will be some class time attributed to this, the emphasis of the task will be on students finding an area of mathematics that interests and resonates with them and then investigating it for themselves.

TOK, CAS and International Mindedness

Mathematics has historically been one of the more "straightforward" subjects. Problems have answers, and it's our job to find the answers. This programme will challenge students to go beyond this way of thinking and consider their learning in a deeper and more profound manner. For example, through considering the nature of mathematical knowledge, its benefits, strengths and weaknesses, or perhaps through engaging with global issues of resources, sustainability or welfare. In summary, students will be encouraged throughout the course to consider the impact of their learning personally, locally and globally.

AIMS OF TH

COURSE

The aims of this programme are to enable students to:

1 Develop a curiosity and enjoyment of mathematics, and appreciate its elegance and power.

2 Develop an understanding of the concepts, principles and nature of mathematics.

3 Communicate mathematics clearly, concisely and confidently in a variety of contexts.

4 Develop logical and creative thinking, and patience and persistence in problem solving to instill confidence in using mathematics.

5 Employ and refine their powers of abstractions and generalization.

6 Take action to apply and transfer skills to alternative situations, to other areas of knowledge and to future developments in their local and global communities.

7 Appreciate how developments in technology and mathematics influence each other.

8 Appreciate the moral, social and ethical questions arising from the work of mathematicians and the applications of mathematics.

9 Appreciate the universality of mathematics and its multicultural, international and historical perspectives.

1 0 Appreciate the contribution of mathematics to other disciplines, and as a particular “area of knowledge” in the TOK course

11 Develop the ability to reflect critically upon their own work and the work of others.

12 Independently and collaboratively extend their understanding of mathematics.

TEACHING AND LEARNING

The diploma programme mathematics course builds in many ways upon the MYP in that it is an inquiry-based course that provides students with opportunities to work in groups or independently, investigate a variety of authentic issues and problems and communicate their ideas with increasing levels of sophistication. It also develops four aspects of mathematical learning that were central to the MYP; Inquiry, Modelling, Technology and proof.

Having come from the MYP programme, Kaisei students will be familiar with ATL skills and their clusters. These will also be key to teaching and learning in the DP course. Furthermore, the idea of conceptual learning is also present. The diploma programme outlines the following 12 concepts to promote mathematical inquiry.

Approximation
Change
Equivalence
Generalization
Modelling
Patterns
Quantity
Relationships
Representation
Space
Systems
Validity

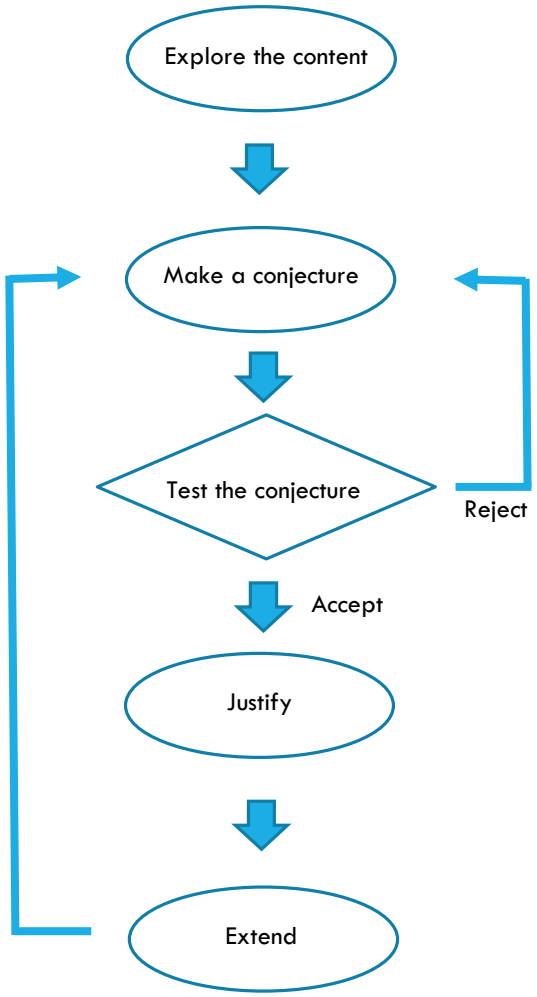
(Explanations for each of these can be found in the appendices)

As students progress through the course, they will be encouraged to inquiry not just about the course content but as learners in general, thinking critically about their own learning, goals and pathways. The skills they practice in class, be that creative/critical thinking, problem solving or communication, will no doubt support them as learners both in and out of the classroom.

TEACHING AND LEARNING

Mathematical Inquiry

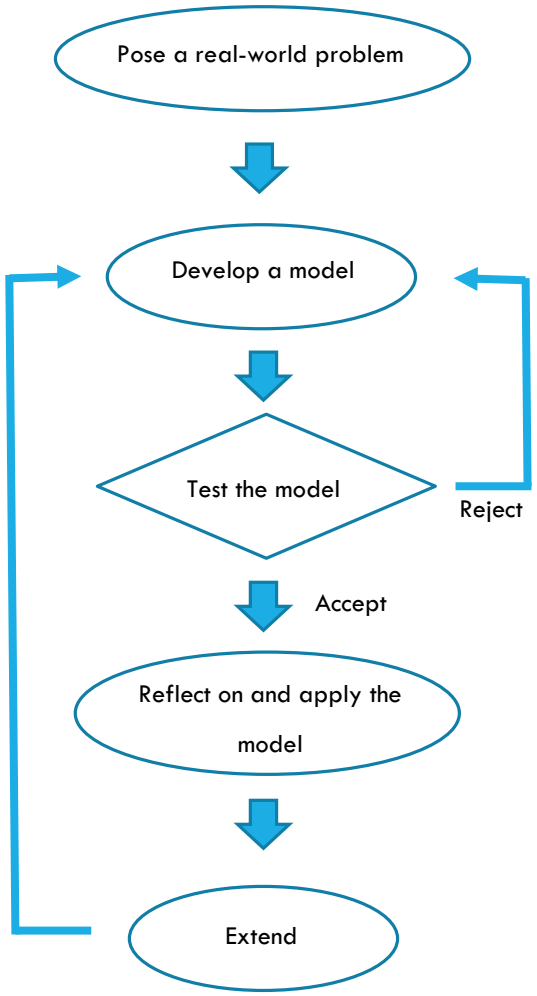
The IB Learner Profile encourages students to learn through experimenting and discovering. As such IB classes will not simply expect students to follow plain instructions but rather actively participate in class, using their own initiative to promote learning for themselves and others. As such teachers will provide students with tasks and problems that allow for such engagement, the process of which is outlined in the diagram below.



TEACHING AND LEARNING

Mathematical Modelling

As a key aspect of the course, students will be expected to express natural phenomena using models. These models will then be used to analyze and draw conclusions which can be used to refine the model or even develop new ones. (See diagram below)



TEACHING AND LEARNING

Using Technology

Technology is a powerful tool. It also an ever-changing and developing tool. As such it is the goal of this course to provide students with appropriate technological fluency, not just to solve problems but to become adept technological learners that can make use of the technology at their disposal to support their own learning, provide clarity in communicating their ideas, or complete powerful calculations in order to efficiently provide deep insight and analysis. Whilst data collection and analysis is a prominent part of the course, it is not the only area in which students will be expected to engage with technology. As such students should be prepared to make use of whatever tools they can, to add insight to a variety of mathematical contexts.

Proof

Proof is an essential aspect of mathematical study. Over the course students will engage with a number of powerful proving methods such as proof by induction or proof by counterexample. While these ideas are mathematical in nature they will be framed to promote wider thinking about communication and justification. Is my argument supported? Is my logic clear and understandable? Is my argument coherent and complete? All of these ideas can be addressed through proof but subsequently applied beyond subject boundaries to promote reasoning skills in all areas.

PRIOR LEARNING

For students undertaking this course of study, it is expected that they are familiar with a number of topics. These topics have been covered throughout the MYP, however, if students are concerned about a particular area they are encouraged to revise this prior to the course and/or seek advice from their mathematics teacher.

Additionally, students are expected to be familiar with standard index (SI) units.

On the following pages is listed a full list of topics that students are expected to have covered prior to the course.

トピック	Contents
Number and Algebra	•Number systems: natural numbers ℕ; integers, ℤ; rationals, ℚ, and irrationals; real numbers, ℝ •SI (Système International) units for mass, time, length and their derived units, eg. speed, area and volume •Rounding, decimal approximations and significant figures, including appreciation of errors •Definition and elementary treatment of absolute value (modulus), a •Use of addition, subtraction, multiplication and division using integers, decimals and fractions, including order of operations •Prime numbers, factors (divisors) and multiples •Greatest common factor (divisor) and least common multiples (HL only) •Simple applications of ratio, percentage and proportion •Manipulation of algebraic expressions, including factorization and expansion •Rearranging formulae •Calculating the numerical value of expressions by substitution •Evaluating exponential expressions with simple positive exponents •Evaluating exponential expressions with rational exponents (HL only) •Use of inequalities, <,≤,>,≥, intervals on the real number line •Simplification of simple expressions involving roots (surds or radicals) •Rationalising the denominator (HL only) •Expression of numbers in standard form•Familiarity with commonly accepted world currencies •Solution of linear equations and inequalities •Solution of quadratic equations and inequalities with rational coefficients (HL only) •Solving systems of linear equations in two variables •Concept and basic notation of sets. Operations on sets: union and intersection •Addition and subtraction of algebraic fractions (HL only).
Functions	•Graphing linear and quadratic functions using technology •Mappings of the elements of one set to another. Illustration by means of sets of ordered pairs, tables, diagrams and graphs.
Geometry and Trigonometry	•Pythagoras' theorem and its converse •Mid-point of a line segment and the distance between two points in the Cartesian plane •Geometric concepts: point, line, plane, angle •Angle measurement in degrees, compass directions •The triangle sum theorem •Right-angle trigonometry, including simple applications for solving triangles •Three-figure bearings •Simple geometric transformations: translation, reflection, rotation, enlargement •The circle, its centre and radius, area and circumference. The terms diameter, arc, sector, chord, tangent and segment• •Perimeter and area of plane figures. Properties of triangles and quadrilaterals, including parallelograms, rhombuses, rectangles, squares, kites and trapezoids; compound shapes

	•Familiarity with three-dimensional shapes (prisms, pyramids, spheres, cylinders and cones) •Volumes and surface areas of cuboids, prisms, cylinders, and compound three-dimensional shapes
Statistics and Probability	•The collection of data and its representation in bar charts, pie charts, pictograms, and line graphs •Obtaining simple statistics from discrete data, including mean, median, mode, range •Calculating probabilities of simple events •Venn diagrams for sorting data •Tree diagrams
Calculus	•Relationship between speed, distance and time

Syllabus Topics

For a more detailed breakdown of topics and concepts, please refer to relevant IB guide.

	Topic	Details	Allotted Time (Approx.)
1	Number and Algebra	1.1 Standard form 1.2 Arithmetic sequences and Series 1.3 Geometric Sequences and Series 1.4 Financial Applications of sequences and series 1.5 Laws of exponents and logarithms 1.6 Simple deductive proof 1.7 Rational exponents 1.8 Sum of infinite convergent sequences 1.9 Binomial Theorem 1.10 **AHL** Counting principles and extended binomial theorem 1.11 **AHL** Partial fractions 1.12 **AHL** Complex numbers and complex plane 1.13 **AHL** Cartesian, polar and exponential form of complex no.s 1.14 **AHL** Complex conjugates, roots and De Moivre's Theorem 1.15 **AHL** Proof by induction, proof by contradiction, proof by counterexample 1.16 **AHL** Systems of linear equations (Solving and analyzing)	39
2	Functions	2.1 Equation of a straight line 2.2 Theory of functions 2.3 Graphing functions 2.4 Key features of graphs 2.5 Composite functions, identity function, inverse functions 2.6 Quadratic functions 2.7 Solving quadratic equations 2.8 Reciprocal functions 2.9 Exponential functions and their graphs 2.10 Using technology to solve equations 2.11 Graph transformations 2.12 **AHL** Polynomials, roots and zeros, factor theorem and remainder theorem 2.13 **AHL** Rational functions 2.14 **AHL** Odd, even and self-inverse functions 2.15 **AHL** Solutions to inequalities both graphically and analytically 2.16 **AHL** Modular graphs (Absolute values)	32
3	Geometry and Trigonometry	3.1 Distance, position, area and volume (3D) 3.2 Sine, Cosine, Tangent, Sine Rule and Cosine rule 3.3 Applications of right angled and non-right angled geometry. 3.4 Radian measure, arc length, area of a sector 3.5 Unit circle 3.6 Pythagorean identity and trig ratios 3.7 Circular functions, transformations and real life contexts 3.8 Solving trigonometric equations 3.9 **AHL** Further trig identities 3.10 **AHL** Compound angle identities 3.11 **AHL** Symmetry of trig functions/graphs 3.12 **AHL** Vectors 3.13 **AHL** scalar product and angles between vectors.	51

		3.14 **AHL** Vector equations in 2 and 3 dimensions 3.15 **AHL** Parallel, intersecting and skew lines 3.16 **AHL** Vector product 3.17 **AHL** Vector equation of planes 3.18 **AHL** Intersections and angles between planes/lines	
4	Statistics and Probability	4.1 Populations, samples and data types 4.2 Presenting and communicating data 4.3 Mean, median, mode, quartiles and interpreting them 4.4 Scatter diagrams and linear correlations 4.5 Trials and outcomes 4.6 Venn diagrams, combine events and conditional probability 4.7 Discrete random variables and their mean and variance 4.8 Binomial distribution 4.9 Normal and inverse normal distributions 4.10 Regression lines 4.11 Conditional probability 4.12 Standardization of normal variables 4.13 **AHL** Bayes Theorem 4.14 **AHL** Continuous probability density functions	33
5	Calculus	5.1 Limits and derivatives 5.2 Increasing and decreasing functions 5.3 Basic derivatives 5.4 Tangents and normal 5.5 Basic integration 5.6 Chain rule 5.7 Second derivatives 5.8 Extrema, optimization and inflexion points 5.9 Kinematic problems 5.10 Reversing the chain rule 5.11 Definite integration 5.12 **AHL** Continuity and higher derivatives 5.13 **AHL** Limits and L'Hopital's rule 5.14 **AHL** Implicit differentiation 5.15 **AHL** Further trig derivatives and partial fractions 5.16 **AHL** Further integration techniques 5.17 **AHL** Areas and volumes 5.18 **AHL** Differential equations 5.19 **AHL** Maclaurin Series	55
	Toolkit and mathematical exploration	Students undertake a piece of personal mathematical research.	30

ASSESSMENT

Assessment is an integral part of teaching and learning. The most important aim of assessment in the DP is that it should support curricular goals and encourage appropriate student learning. Both external and internal assessments are used in the DP. IB examiners mark work produced for external assessment, while work produced for internal assessment is marked by teachers and externally moderated by the IB.

評価要素	配点比率
External Assessment（5 Hours）	80％
Paper 1 （2 Hours） No technology allowed. (110 marks) Section A Compulsory short-response questions based on the syllabus. Section B Compulsory extended-response questions based on the syllabus.	30％
Paper 2 (2 Hours) Technology required. (110 marks) Section A Compulsory short-response questions based on the syllabus. Section B Compulsory extended-response questions based on the syllabus.	30％
Paper 3 (1 Hour) Technology required. (55 marks) Two compulsory extended response problem-solving questions.	20％
Internal Assessment This component is internally assessed by the teacher and externally moderated by the IB at the end of the course. **Mathematical exploration** Internal assessment in mathematics is an individual exploration. This is a piece of written work that involves investigating an area of mathematics. (20 marks)	20％

COMMAND TERMS

Command Terms in Mathematics

Students are to have a clear understanding of the terms used in exams to set tasks and ask questions. On the following pages is a table that lists the words in both English and Japanese and what is meant by each word. Students should take time to become familiar with this list.

Command Terms for Mathematics

計算しなさい	Calculate	作業の過程を適切に示しながら、答えとなる数値を求めなさい。
コメントしなさい	Comment	与えられた記述または計算結果に基づき、見解を述べなさい。
比較しなさい	Compare	2つ（またはそれ以上）の事柄または状況の類似点について、常に双方（またはすべて）について言及しながら、説明しなさい。
比較・対比しなさい	Compare and contrast	2つ（またはそれ以上）の事柄または状況の類似点および相違点について、常に双方（またはすべて）について言及しながら説明しなさい。
作成しなさい	Construct	図表形式または論理形式で情報を示しなさい。
対比しなさい	Contrast	2つ（またはそれ以上）の事柄または状況の相違点について、常に双方（またはすべて）について言及しながら、説明しなさい。
推論しなさい	Deduce	与えられた情報から結論を導き出しなさい。
論証しなさい	Demonstrate	具体例や実際の応用例を挙げながら、推論または根拠に基づいて明らかにしなさい。
詳しく述べなさい	Describe	詳細に述べなさい。
決定しなさい	Determine	考えられる唯一の答えを求めなさい。
微分しなさい	Differentiate	関数の導関数を求めなさい。
区別しなさい	Distinguish	2つまたはそれ以上の概念または事柄の相違点を明確にしなさい。
描きなさい、作図しなさい	Draw	鉛筆を用いて、名称がつけられた正確な図またはグラフとして表しなさい。直線には直定規を用いること。図表は一定の縮尺で描きなさい。グラフは（該当する場合）正確に点を書き入れ、直線または滑らかな曲線でつなぎなさい。
概算しなさい、見積もりなさい	Estimate	およその値を求めなさい。
説明しなさい	Explain	理由や要因などを詳しく述べなさい。
求めなさい	Find	作業の過程を適切に示しながら答えを得なさい。
前問の結果を用いて	Hence	前問の結果を利用し、要求されている結果を得なさい。
必要ならば前問の結果を用いて	Hence or otherwise	前問の結果を利用してもよいが、それ以外の方法を用いてもよい。
特定しなさい	Identify	数ある可能性の中から答えを確定しなさい。
積分しなさい	Integrate	関数の積分を求めなさい。
解釈しなさい	Interpret	与えられた情報から傾向をつかんで結論を引き出すため、知識と理解を用いなさい。
調べなさい	Investigate	事実を立証し新たな結論に到達するため、観測、調査または詳細かつ体系的な検証を行うこと。
正当化しなさい	Justify	答えや結論を裏づける妥当な理由や根拠を述べなさい。
名称をつけなさい	Label	図表に名称をつけなさい。
列挙しなさい	List	説明をつけ加えずに、簡潔な答えを述べなさい。
プロットしなさい	Plot	図表上に点の位置を書き入れなさい。
予測しなさい	Predict	予想されている結果を示しなさい。
証明しなさい	Prove	形式的な推論の積み重ねによって、要求されている結果を得なさい。

示しなさい。	Show	計算過程や結果の導出結果を示しなさい。
～であることを示しなさい	Show that	証明の手順を踏まず（場合によっては与えられた情報を用いて）要求された結果を出しなさい。「～であることを示しなさい」という問題は通常、電卓は必要ありません。
解きなさい	Solve	代数、計算、グラフのいずれか、またはいずれかの組み合わせを用いて答えを求めなさい。
提案しなさい	Suggest	解決策、仮設、またはその他の考えられる答えを示しなさい。
述べなさい	State	説明または計算することなしに、特定の名称、数値、またはその他の簡潔な答えを示しなさい。
略図を描きなさい、大まかな図やグラフを描きなさい	Sketch	（必要に応じて名称をつけ）図表またはグラフで表しなさい。略図は、求められる形または関係の概観を示し、特徴を表したものでなければなりません。
確かめなさい	Verify	結果の正当性を示す根拠鵜を提示しなさい。
書き出しなさい	Write down	主に情報を抜き出すことによって答えを得なさい。計算はほとんど必要なく、過程を記す必要もありません。

Appenix

Approximation	This concept refers to a quantity or a representation which is nearly but not exactly correct.
Change	This concept refers to a variation in size, amount or behaviour.
Equivalence	This concept refers to the state of being identically equal or interchangeable, applied to statements, quantities or expressions.
Generalization	This concept refers to a general statement made on the basis of specific examples.
Modelling	This concept refers to the way in which mathematics can be used to represent the real world.
Patterns	This concept refers to the underlying order, regularity or predictability of the elements of a mathematical system.
Quantity	This concept refers to an amount or number.
Relationships	This concept refers to the connection between quantities, properties or concepts; these connections may be expressed as models, rules or statements. Relationships provide opportunities for students to explore patterns in the world around them.
Representation	This concept refers to using words, formulae, diagrams, tables, charts, graphs and models to represent mathematical information.
Space	This concept refers to the frame of geometrical dimensions describing an entity.
Systems	This concept refers to groups of interrelated elements.
Validity	This concept refers to using well-founded, logical mathematics to come to a true and accurate conclusion or a reasonable interpretation of results.